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CMBR anisotropy in the framework of cosmological extrapolation of MOND

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Abstract. A modified gravity involving a critical acceleration, as empirically established at galactic scales and successfully tested by data on supernovae of type Ia, can fit the measured multipole spectrum of anisotropy in the cosmic microwave background radiation, so that a dark sector of Universe is constructively mimicked as caused by the dynamics beyond the general relativity. Physical consequences, verifiable predictions and falsifiable issues are listed and discussed.

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1. Introduction

The general relativity as the theory of gravitation is triumphantly tested in "classical experiments" [1] on

- the deflection of light by the Sun,
- the perihelion precession of Mercury,
- the gravitational redshift of electromagnetic radiation,
- the time delay of signal from satellites due to the curved space-time, and
- the gyroscope precession during the orbital motion around the Earth as caused by the spatial curvature [2, 3].

In addition, an expanding Universe is the prediction being inherent for the general relativity. Sure, the general relativity conceptually is the perfect theory of classic gravity. In this respect, we usually expect that it is valid, indeed, until effects of quantum gravity would be essential at Planckian scales of energy that is unreachable in practice. However, such the point of view, perhaps, is actually broken:

The general relativity itself gives us a brilliant tool in order to search for indications, which signalize on breaking down its validity: while observing a motion by inertia, we get a curvature of space-time, which can be inserted into Einstein equations, that yields a tensor of energy-momentum for an appropriate substance, and if properties of the substance are mysterious and unpredictable, then we get a hint for suspecting of the incorrect description for the nature. This is exactly the case of hypothetic dark matter (see, for instance, review in [4]): in the framework of general relativity, it should be inevitably introduced as a transparent pressureless substance dynamically isolated from the ordinary visible matter made of known, well studied particles, except the interaction via the gravity, so that properties of dark matter are artificially tuned. This tuning has various aspects.

First, rotational curves in disc-like galaxies, i.e. dependencies of rotation velocities of stars versus a distance to the galaxy center, if described by the law of Newtonian gravity, requires the introduction of dark matter with a tuned spatial distribution. Unexpectedly, the dark matter halo is inessential in regions, wherein the gravitational acceleration caused by the visible matter, is greater than a critical value $\tilde{g}_0$, while the halo starts to dominate in regions, wherein the acceleration by the visible matter is less than $\tilde{g}_0$ [4, 5, 6, 7]. That was M. Milgrom who first introduced the critical acceleration $\tilde{g}_0$ in the description of rotational curves [8]. It is spectacular that the critical acceleration is universal: it does not depend on a genesis of disk-like galaxy, and it is the same for any studied disc-like galaxy. Unbelievably, an amount and spatial distribution of dark matter is tuned to the amount and distribution of visible matter in order to form in cosmic collisions the dark halos in disc-like galaxies with the same universal critical acceleration. In the framework of general relativity, there is no straightforward dynamical mechanism for a deduction of such the universal acceleration. Anyway, the deduction looks to be very artificial, most probably, it certainly could be the fine tuning.

The critical acceleration subtly binds the dark matter to the visible matter. If this relation is dynamical, then it is not due to Newtonian gravity, that does not include any critical acceleration. Further, in deep regions of dark halo dominance, the rotation velocities tend to constant values v_0, that empirically satisfy the baryonic Tully–Fisher law [9]:

$$v_0^4 = GM\tilde{g}_0, \tag{1}$$

where M stands for the mass of visible matter in disc-like galaxy, G denotes the Newton constant. Again, the dark matter halo is tuned, so that the star motion within the halo strictly correlates with the usual visible matter, while the constant $\tilde{g}_0$ is universal [10]. Finally, in order to complete the first item of argumentation, features in distributions of visible matter, no doubt, are imprinted in rotational curves even in regions of dark matter dominance [4], hence, features of dark matter distributions are tuned to the visible matter, though we have no dynamical reasons for such the correlations in the framework of general relativity. Moreover, the morphology of spatial distributions is absolutely different for the baryonic and dark matter in disc-like galaxies: an exponentially falling central bulge and thin disc of stars and gas in contrast to power-law decline of dark spherical halo. Thus, the universal critical acceleration is the mysterious quantity for the general relativity, that cannot be predicted, while its notion emerged empirically. The critical acceleration $\tilde{g}_0$ reveals the fine tuning of hypothetic dark matter to the ordinary visible matter.

Second, in cosmology with the observed accelerated expansion of Universe by data on a dependence of brightness of type Ia supernovae versus the red shift [11, 12, 13, 14, 15], the general relativity was have to introduce the extended dark sector, which includes a homogeneous dark energy in addition to the nonhomogeneous dark matter. In the simplest case, the dark energy can be represented by the cosmological constant, otherwise it should be described by a homogeneous fluid X with the state parameter w_X being the ratio of pressure p_X to energy density ρ_X, $w_X = p_X/\rho_X$ close to vacuum value of -1, in contrast to the pressureless dark matter with $w_{\mathrm{DM}} = 0$. Evidently, the nature of dark matter and dark energy is very different. But surprisingly, the energy density of dark energy, or the value of cosmological constant Λ, is finely tuned to the critical acceleration [4], so that

$$G\rho_X \sim \Lambda \sim \tilde{g}_0^2. \tag{2}$$

Therefore, the dark energy should be inherently connected to the unexpectedly correlated dynamics of dark matter and ordinary matter. But the coincidence of (2) is mysterious for the general relativity.

Nevertheless, the general relativity applied to the cosmos still looks formally viable in the form of concordance model: the ordinary matter balanced with the cold dark matter (CDM) and cosmological constant Λ in the flat space, the ΛCDM variant with a spatial curvature compatible with zero in limits of uncertainties. Moreover, there are two important successes in the model: 1) a correct fitting for observed anisotropy of cosmic microwave background radiation (CMBR) [16] that becomes possible due to a

tuned amount of non-baryonic dark matter, and 2) an appropriate baryon to photon ratio consistent with a current status of big bang nucleosynthesis (BBN) (see review by Fields B D, Sarkar S in [17]).

Indeed, the anisotropy of CMBR is suppressed by 5 orders of magnitude with respect to the CMBR temperature, and it is caused by a propagation of sound waves in a hot photon-electron-baryon medium up to a moment, when the electrons bind to nuclei to form neutral transparent gas. The snapshot of Universe at the time of decoupling of photons evolves to us, and it represents acoustic peaks in the following multipole spectrum of temperature fluctuations

$$\langle \Delta T(\boldsymbol{n}_1)\Delta T(\boldsymbol{n}_2)\rangle = \sum_l \frac{2l+1}{4\pi}\, C_l\, P_l(\boldsymbol{n}_1\cdot\boldsymbol{n}_2), \tag{3}$$

where $\boldsymbol{n}_{1,2}$ denote directions in the celestial sphere, P_l are Legendre polynomials of multipole number l. The spectrum, i.e. C_l, depends on

- the Universe evolution,
- a primary spectrum of inhomogeneity, and
- the propagation of inhomogeneity during the evolution.

Then, the Universe evolution is well described by ΛCMD [16] and it can be extrapolated to the age of Universe, when the snapshot of CMBR was done, i.e about 380 thousands years after the big bang. The primary spectrum of inhomogeneity is suggested to be close to the Harison–Zeldovich distribution of so called "no-scale" limit at a spectral index $n_s(k) = 1$:

$$\left\langle \frac{\delta\rho^2(0)}{\rho^2} \right\rangle = A \int \left(\frac{k}{k_0}\right)^{n_s(k)-1} \mathrm{d}\ln \mathrm{k}, \tag{4}$$

where $\delta\rho(\boldsymbol{r}) = \rho(\boldsymbol{r}) - \rho$ denotes a contrast of energy density at comoving coordinate, k is a wave vector conjugated to the comoving coordinate, A stands for an amplitude at a reference value of k_0. The spectral index and amplitude are subjects to fit the observed spectrum of C_l. Finally, the propagation of inhomogeneity includes the sound and further smearing of waves by the gravitation. The concordance model of cosmology in the framework of general relativity well fits C_l with the flat space and tuned amount of dark components [16]: relative fractions of dark matter $\Omega_{\rm DM} \approx 20\%$ and dark energy in the approximation of cosmological constant $\Omega_\Lambda \approx 76\%$. The baryonic matter composes only $\Omega_b \approx 4\%$. This value is dictated by heights of distinct initial three acoustic peaks in the multipole spectrum, C_l. That is the dark matter fraction, which regulates the relative heights and positions of peaks up to small variations due to the parameters of primary spectrum of inhomogeneity.

Next, the amount of baryonic matter and the temperature of CMBR fixes the baryon-to-photon ratio of densities $\eta_b = n_b/n_\gamma$, that is the only free parameter in calculation of elements abundance during the big bang nucleosynthesis. The current state of measurements of elements abundance extrapolated to the primary abundance is compatible with η_b extracted from the ΛCDM fit of CMBR anisotropy [17].

Thus, the success of concordance model stimulates direct searches for an appropriate heavy dark matter particle at colliders and underground big-volume experiments, sensitive to suppressed, but non-zero cross sections of dark matter interaction with the ordinary matter.

However, even a discovery of candidate for the dark matter particle would not withdraw the problem of ad hoc tuning of dark sector. Moreover, it would sharpen the need to search for the dynamical reasons causing the adjustment of dark matter, i.e to look beyond the general relativity.

A model of gravity involving the critical acceleration should naturally include both empirical laws such as the Tully-Fisher relation at the galactic scales and correct descriptions of Universe evolution, observed features of CMBR, large scale structure and elements abundance, so that the model would give a successful approach being alternative to the general relativity in cosmology, of course. At galactic scales, M. Milgrom invented the modified Newtonian dynamics (MOND) [8] stating the gravitational acceleration $\boldsymbol{g}$

$$\boldsymbol{g}\,\zeta(g/\tilde{g}_0) = -\boldsymbol{\nabla}\phi_M, \qquad \zeta(y) = \frac{y}{\sqrt{1+y^2}}, \tag{5}$$

where the critical acceleration extracted from the modern analysis of rotational curves is given by $\tilde{g}_0 = (1.24 \pm 0.14) \times 10^{-10}\ m/s^2$ [10], while ϕ_M denotes the Newtonian gravitational potential of ordinary matter, satisfying $\boldsymbol{\nabla}^2\phi_M = -4\pi\, G\, \rho_M$. At $g/\tilde{g}_0 \gg 1$, we get the Newtonian limit of gravitational force‡, while at $g/\tilde{g}_0 \ll 1$ the Tully-Fisher law is satisfied by construction. Note that (5) successfully predicts the rotational curves by the given distribution of visible matter§ with appropriate imprints of its features, see review in [4].

However, the straightforward insertion of (5) into the dynamics at cosmological scales would results in an inconsistent distortion of vacuum homogeneity during the evolution of Universe, for instance, i.e. in the vacuum instability as was shown in [18], wherein authors offered to introduce the cosmological behavior of critical acceleration in the form of

$$\tilde{g}_0 \mapsto g_0 = g_0'\,|\boldsymbol{x}|, \tag{6}$$

where the distance is determined by comoving coordinate $\boldsymbol{r}$ and scale factor of Universe expansion $a(t)$, so that $\boldsymbol{x} = a(t)\,\boldsymbol{r}$. Then, the homogeneous cosmology with the gravity law modified at accelerations below the critical value of $\tilde{g}_0$ is consistent, that constitutes the cosmological extrapolation of MOND [18]. The cosmological regime is matched to MOND at a size of large scale structure $|\boldsymbol{x}|_{\rm lss}$, i.e. at the characteristic scale of

‡ Sub-leading terms are suppressed, so that the force at the Earth and in the Solar system is not distinguishable from the Newtonian one.

§ The only parameter of fitting the rotational curves within MOND is the light-to-mass ratio, which strictly correlates with astrophisical expectations for given galaxies. Moreover, in gas-rich galaxies this uncertainty is absent, that means the MOND predicts rotational curves with no adjustment of any parameters, see details in [4].

inhomogeneity, that is related to the acoustic scale and sound horizon in the baryon-electron-photon plasma (see details in [18]).

In the framework of cosmological extrapolation of MOND with the interpolation function $\zeta(y)$ in (5) the gravity equations for the evolution of homogeneous and isotropic Universe can be written in the form [18]

$$\left(R^\nu_\mu\xi^\mu\xi_\nu\right)^4 = \Big(\left(R^\nu_\mu\xi^\mu\xi_\nu\right)^2 + \left(K^\nu_\mu\xi^\mu\xi_\nu\right)^2\Big)\left(\bar{R}^\nu_\mu\xi^\mu\xi_\nu\right)^2, \tag{7}$$

in terms of Ricci tensor $R_{\mu\nu}$ for the metric $\eta_{\mu\nu}$ and 4-vector in the direction of cosmological time $\xi^\mu = (1,\mathbf{0})$, wherein the matter energy-momentum tensor $T_{\mu\nu}$ defines

$$\bar{R}^\nu_\mu[\eta] = 8\pi G\left(T^\nu_\mu - \tfrac{1}{2}\eta^\nu_\mu T\right), \tag{8}$$

while the extra tensor of curvature K^ν_μ is the Ricci tensor of de Sitter space-time being both homogeneous in time and space as well as isotropic

$$K^\nu_\mu = 3g_0'\eta^\nu_\mu. \tag{9}$$

In addition to (7) the conservation of energy-momentum $\nabla_\mu T^\mu_\nu = 0$ is hold, of course.

Then, in the limit of general relativity we put $\left(R^\nu_\mu\xi^\mu\xi_\nu\right)^2 \gg \left(K^\nu_\mu\xi^\mu\xi_\nu\right)^2$, and we find $R^\nu_\mu \approx \bar{R}^\nu_\mu$, that results in the Einstein equations

$$R^\nu_\mu = 8\pi G\left(T^\nu_\mu - \tfrac{1}{2}\eta^\nu_\mu T\right). \tag{10}$$

In the limit of $\left(R^\nu_\mu\xi^\mu\xi_\nu\right)^2 \ll \left(K^\nu_\mu\xi^\mu\xi_\nu\right)^2$ we get the modified evolution of Universe, effective at present.

It is important that parameterizing the size of large scale structure by $|\boldsymbol{x}|_{\rm lss} \sim \lambda^2/H_0$ at $g_0' \sim H_0^2/\lambda$ and moderate value of $\lambda \sim \frac{1}{7}$, we find the simplest solution for the coincidence problem, because the Milgrom acceleration $\tilde{g}_0 \sim \lambda H_0$ becomes close to the scale of cosmological constant $\Lambda \sim H_0^2$. Moreover, we will see that the dark matter fraction of energy is also regulated by the value of λ.

Eq. (7) successfully fits the evolution of Universe measured by observing the brightness of type Ia supernovae versus the red shift [18]. So, the stellar magnitude

$$\mu = \mu_{abs} + 5\log_{10} d_L(z) + 25, \tag{11}$$

depends on the photometric distance d_L (in Mpc), determined by the Hubble constant evolution $H(z) = \dot{a}/a$,

$$d_L(z) = (1+z)\int\limits_0^z \frac{dz}{H(z)}, \tag{12}$$

where μ_{abs} is an absolute stellar magnitude of light source at the distance of 10 pc. We show the Hubble diagram for the type Ia Supernovae in Fig. 1. The mean deviation squared per degree of freedom gives $\chi^2/\text{d.o.f.} = 1.03$ for our fit with the following assignment of parameters:

$$\begin{aligned} q_0 &= -0.853, & z_t &= 0.375, \\ h &= 0.71, & \Omega_b &= 0.115, \end{aligned} \tag{13}$$

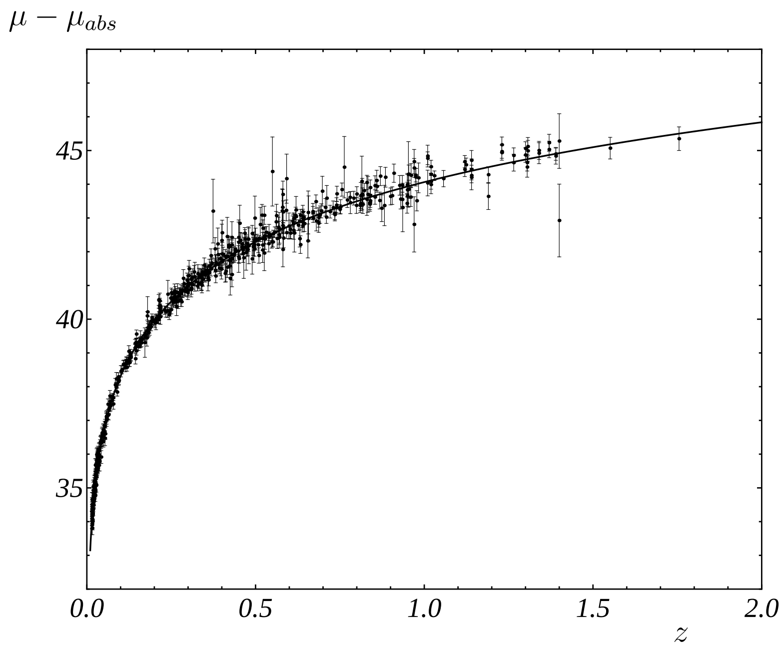

Figure 1. The magnitude of supernova luminosity versus the redshift z. The data with error bars are taken from the Union2 collection [15] and the results of Hubble Space Telescope on the type Ia supernovae [12]. The curve represents our fit in the framework of modified cosmology.

where q_0 determines the deceleration parameter at red sift $z = 0$, $q(z) = -\ddot{a}/aH^2(z)$, z_t stands for the red shift, when the acceleration is equal to zero, h parameterizes the Hubble rate at $z = 0$ via $H_0 = h \cdot 100\,\mathrm{km/s} \cdot \mathrm{Mpc}^{-1}$. Accepting the prescription of

$$g_0' = K_0 H_0^2, \tag{14}$$

we can find that

$$q_0 = \frac{1}{2}\, K_0 \Omega_b \left((1 + z_t)^3 - 1\right), \tag{15}$$

when the energy density determined by the cosmological constant, is given by the energy budget of Universe, $\Omega_\Lambda = 1 - \Omega_b$. The same values of deceleration, q_0, and red shift of transition from the deceleration to acceleration, z_t, could be obtained in concordance model of ΛCDM‖ at

$$\frac{\bar{\Omega}_M}{\bar{\Omega}_b} = -\frac{K_0}{q_0}. \tag{16}$$

Therefore, the modified gravity with the critical acceleration, i.e. at nonzero $K_0 \approx 7.9$, determines the ratio of matter to baryon fractions of energy, so that at $q_0 \sim -1$ we get $\bar{\Omega}_M/\bar{\Omega}_b \sim K_0$, which correlates with the scale of large scale structure, considered above, $K_0 \sim 1/\lambda$.

‖ The ΛCDM parameters are marked by bars.

Note that the quality of fit is not sensitive to valuable variations of baryonic fraction Ω_b, while it was extracted from the appropriate value of sound horizon

$$r_s(z) = \int\limits_0^{t(z)} c_s\, dt, \tag{17}$$

where the speed of sound is determined by the baryon-to-photon ratio of energy R, so that

$$c_s = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{1+R}}, \qquad R = \frac{3}{4}\frac{\rho_b}{\rho_\gamma} = \frac{3}{4}\frac{\Omega_b}{(1+z)\Omega_\gamma}. \tag{18}$$

The value of sound horizon at $z = 0.2$ and $z = 0.35$ was extracted from baryonic acoustic oscillations (BAO) [19], which, in the case of baryon matter only, favor for the enhanced estimate of Ω_b shown above. The same conclusion follows from the calculation of acoustic scale in the spectrum of CMBR anisotropy [16],

$$l_A = \frac{\pi d_L(z_*)}{(1+z_*)\, r_s(z_*)}, \tag{19}$$

here z_* is the redshift of decoupling, when due to the recombination of electrons with protons the medium becomes transparent for photons (see analytical approximations for z_* in terms of baryonic density, matter density and Hubble constant in [20].). WMAP gives $l_A = 302.69 \pm 0.76$, while we deduce $l_A = 302.5$ [18] compatible with the uncertainty of measurement. Hence, we expect that scale features of CBMR anisotropy could be fitted in the framework of cosmological extrapolation of MOND.

In present paper we describe the procedure of fitting the CMBR anisotropy spectrum with the model of modified gravity and point to accepted approximations in Section 2. Some actual problems associated with the theory and phenomenology of our model of modified gravity, are considered in Section 3. We present an itemized discussion of model verification and falsification in Section 4.

2. Fitting the CMBR anisotropy

The tool for the calculation of multipole spectrum of CMBR anisotropy [21, 22, 23] operates with the Friedmann equation, which is not valid in the framework of cosmological extrapolation of MOND. Nevertheless, we can integrate out the dynamical equations of our model in order to extract the Hubble rate at any red shift and to parameterize it with a mimic dark energy contribution in addition to the fraction of baryonic matter. Indeed, exploring the general relativity in the case of baryonic matter and dark energy in the isotropic homogeneous curved space we get

$$\frac{H^2}{H_0^2} = \frac{\Omega_b}{a^3} + \frac{\Omega_k}{a^2} + \Omega_X(a), \tag{20}$$

$$-\frac{2\ddot{a}/a}{H_0^2} = \frac{\Omega_b}{a^3} + \Omega_X(a)(1+3w_X(a)), \tag{21}$$

where Ω_k stands for the contribution of space curvature, and the energy budget holds $\Omega_b + \Omega_X(1) + \Omega_k = 1$. Excluding $\Omega_X(a)$, we get the expression for the dark equation of state

$$w_X(a) = -\frac{1}{3}\left(1 + \frac{2\frac{\ddot{a}}{aH_0^2} + \frac{\Omega_b}{a^3}}{\frac{H^2}{H_0^2} - \frac{\Omega_b}{a^3} - \frac{\Omega_k}{a^2}}\right). \tag{22}$$

Here we insert the expression for the acceleration that follows from eq. (7) for the modified gravity including the actual values of energy fractions for baryons and vacuum, Ω_b and Ω_Λ, respectively,

$$\frac{\ddot{a}}{aH_0^2} = \left(\Omega_\Lambda - \frac{\Omega_b}{2a^3}\right)\frac{1}{\sqrt{2}}\sqrt{1 + \sqrt{1 + \frac{(2g_0'/H_0^2)^2}{\left(\Omega_\Lambda - \frac{\Omega_b}{2a^3}\right)^2}}}. \tag{23}$$

Note that the red shift of transition from the acceleration to deceleration of Universe, z_t is related to the fraction of cosmological constant due to

$$\Omega_\Lambda = \frac{1}{2}\,\Omega_b(1+z_t)^3.$$

Again, eq.(23) clearly shows that putting g_0' equal to zero, we get the dynamics of general relativity, otherwise near $\ddot{a} = 0$ the dynamics enters the region of strong dominance of modification and the greatest deviation from the general relativity, as it is actual at present.

Integrating out (23) at the initial condition $\dot{a}/a(t = t_0) = H_0$, we obtain¶ the scaling quantity H/H_0 required for the complete definition of r.h.s. in (22). Fig. 2 shows the behavior of w_X versus the scale factor a as we have calculated in the cosmological extrapolation of MOND with parameters listed in (13) at $\Omega_k = 0$. Small variations of the spatial curvature in limits $|\Omega_k| < 0.02$, deceleration parameter q_0, transitional red shift z_t and baryonic fraction Ω_b within 10% lead to negligible changes, which are only just visible in the figure. We emphasize that the modification of gravity predicts the very specific dependence of equation of state for the dark energy, that we will discuss in Section 4.

After the definition of referenced homogeneous evolution of Universe under the modified gravity, we can use the standard tool for the calculation of CMBR anisotropy spectrum [21, 22, 23]. However, in this way the propagation and smearing of sound waves would be described in the framework of general relativity with no dark matter, i.e. at $\Omega_{\rm DM} = 0$, while we have to modify this procedure in accordance with a structure formation under the modified law at accelerations below the critical one. The problem is that such the modification of perturbation transfer function is not linear, and the appropriate machinery of calculation is not yet developed. That is missing point of our consideration, of course. Nevertheless, the extensive usages of tool have shown that the influence of dark matter on the spectrum is reduced to relative enhancement of third

¶ In practice, we use the scaling variable $\tau = tH_0$, that completely covers the differential equations under consideration.

peak, whereas this enhancement is due to enforcing the gravity. So, since the modified gravity produces the very similar effect of enforcing the gravity, we can expect that it could results in the same fine feature as concerns for the enlarging the third peak.

In this respect, we have to emphasize that a complete axiomatic approach based on a formulation of action for a modified gravity in terms of given extended set of gravitational fields has got the advantage in calculating of various predictions including the CMBR anisotropy. So, we can mention the following fully relativistic schemes (more examples and references find in [4]):

- Bekenstein's theory of tensor-vector-scalar (TeVeS) gravitational fields [24] involving Maxwellian vector field and reproducing the critical acceleration in the case of isolated gravitational source, equivalent to MOND,
- Moffat's modified gravity [25, 26, 27], giving the approximation of gravity law similar to the MOND,
- generalized TeVeS theories [28] with non-Maxwellian vector field.

However, first, these theories include dark gravitational fields actually replacing the dark matter that looks like a refinement of problem. Second, all of them have the strict theoretical illness: there are configurations with unlimited, infinite negative energy, that leads to instability of physically sensible solutions (see details and references in [4]). Third, the critical acceleration is still introduced ad hoc with no reasonable connection to the present Hubble rate or the cosmological constant.

So, we prefer for the phenomenological approach, which does not introduce new artificial and heuristic notions. In this way, we can investigate the role of critical acceleration in the modified gravity by studying various phenomena step by step in order to find fundamental features and differences from the general relativity.

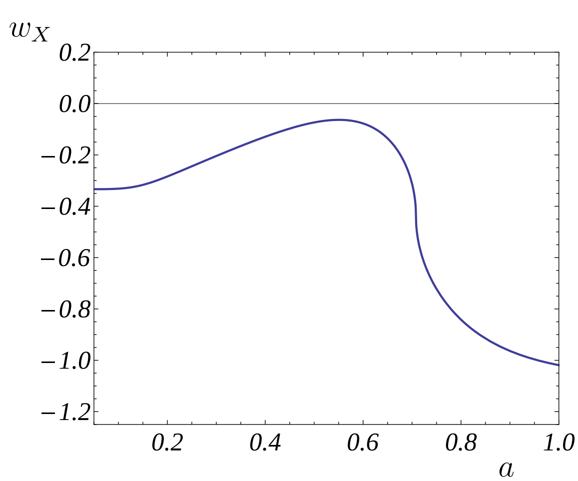

Figure 2. The equation of state w_X versus the scale factor a in the model of modified gravity involving the critical acceleration.

At present, we try to fit the spectrum of CMBR anisotropy by using the modified evolution of homogeneous Universe and optimizing the primary spectrum of inhomogeneity, which develops as the sound smeared by ordinary gravity. So, we suggest that a modification of smearing will not be very crucial for the main features of spectrum.

The results of such the fitting are presented in Figs. 3–5. First, we study the fit of WMAP data [16] with the Harrison–Zeldovich primary spectrum (HS) at $n_s = 1$, which is shown in Fig. 3 by dashed line. It is spectacular that the no-scale HS prescription correctly reproduces the angular scale of multipole momentum, i.e. the position and profile of first acoustic peak. This feature is obtained due to the correct adjustment of this scale by the sound horizon in the case of no dark matter [18] as mentioned in the Introduction. Then, we find that the running of spectral index $n_s(k) = n_s^{(0)} + n'_s \ln k/k_0$ leads to suitable description of both, first and second acoustic peaks in the modified

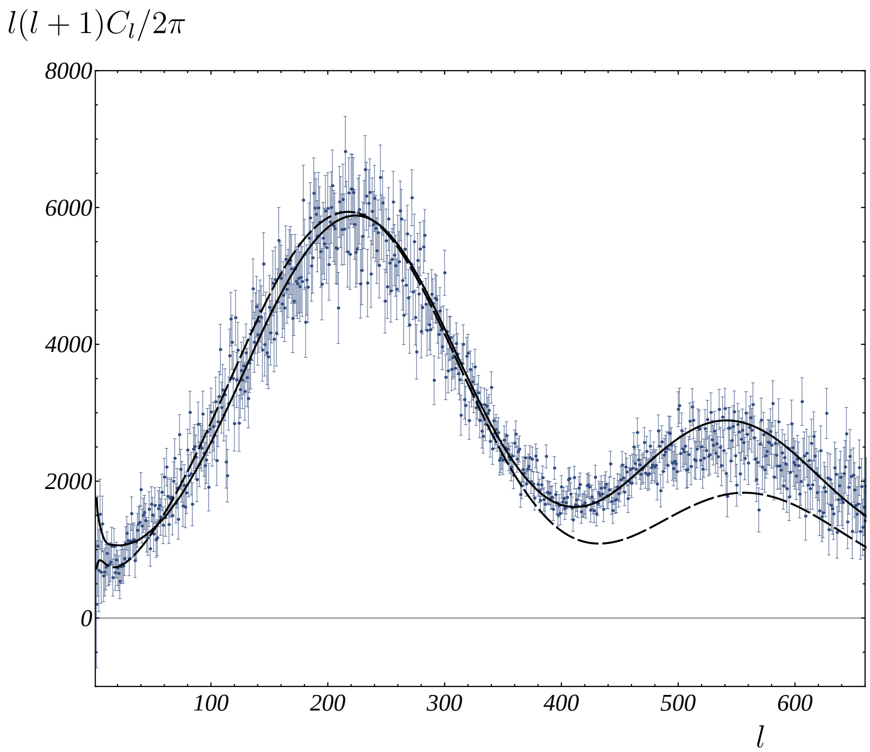

Figure 3. The spectrum of CMBR anisotropy calculated in the model of modified gravity, $l(l+1)C_l/2\pi$ (in μK^2) versus the multipole number l, in comaprison with WMAP data [16]. The dashed line represents the Harrison–Zeldovich approximation, while the solid curve corresponds to the running spectral index (as described in the text).

cosmology with no dark matter. Setting parameters equal to the following values+:

$$\begin{array}{rclrcl} n_s^{(0)} & = & 1.625, & n_s' & = & 0.24, \\ A & = & 5.5\times 10^{-9}, & \Omega_k & = & -0.015, \\ z_{re} & = & 23, & k_0 & = & 0.04\ \mathrm{Mpc}^{-1}, \end{array} \tag{24}$$

we get the fit shown by solid line in Fig. 3. Set (24) needs comments.

First, the running of spectral index allows to adjust the relative height of second acoustic peak in the spectrum*. This running is dynamically essential and numerically significant. Moreover, because of sizable value of slope of spectral index, we expect that the approximation neglecting the higher orders of expansion versus the logarithm of comoving wave vector $\ln k$, would be inaccurate at large intervals of multipole moment.

Second, we introduce a small spatial curvature in order to adjust the position of peaks, which have been displaced under the strong running of spectral index. Then, the spatial curvature essentially improves the quality of fit. Remember, that such the low value of spatial curvature is consistent with the equation of state we have deduce from the modified gravity♯. Moreover, acceleration $\ddot{a}$ does not involve the spatial curvature because of its specific value of state parameter $w_k = -\frac{1}{3}$, giving $\rho_k + 3p_k = 0$. We note also that a nonzero value of spatial curavture obeys the scaling $|K_0^2\Omega_k| \sim 1$, that is a feature consistent with our solution or treatment of coincidence problem (see the Introduction).

Third, the red shift of reionization z_{re}, when inhomogeneities of cold gas are contacted due to the gravity in order to form stars, refers to the heating of gas, that transforms it to plasma again. It causes a reduction of intensity of radiation passing through the hot secondary plasma. So, the amplitude of primary CMBR, A correlates with the red shift of reionization. In MOND with the enhanced gravity at the stage of diluted gas, one expects that the star formation starts early than in the general relativity [4], hence, we try to improve the fit quality by enlarging the red shift of reionization††. The corresponding improvement of mean deviation squared is equal to -0.11 per degree of freedom. The optical depth is shifted from 0.9 to 0.78. So, we refer this adjustment as fine effect beyond a sole significance, as well as slow variation of pivot wave vector k_0 from 0.05 to 0.04 Mpc^{-1}.

Thus, in the simplest way of modifying the background cosmology by the cosmological extrapolation of MOND, we find that WMAP data [16] can be fitted at the following mean deviation squared per degree of freedom:

$$\chi^2/\mathrm{d.o.f.} = 1.34, \tag{25}$$

+ We list the quantities, which we change from default values assigned in the tool [21, 22, 23]. Other quantities have been set to its standard prescriptions.

* The opportunity to fit the second peak in the model with suppressed dark matter was considered in [29].

♯ Note that at $a \to 0$ the equation of state for the mimic dark energy tends to $w_X(0) \approx -\frac{1}{3}$, which points to a possibility of nonzero spatial curvature.

†† We set z_{re} via "a sizable change": the typical value of $z_{re} = 11$ in ΛCDM has been enlarged twice for a distinguishability with no strict reasons or prerequisites.

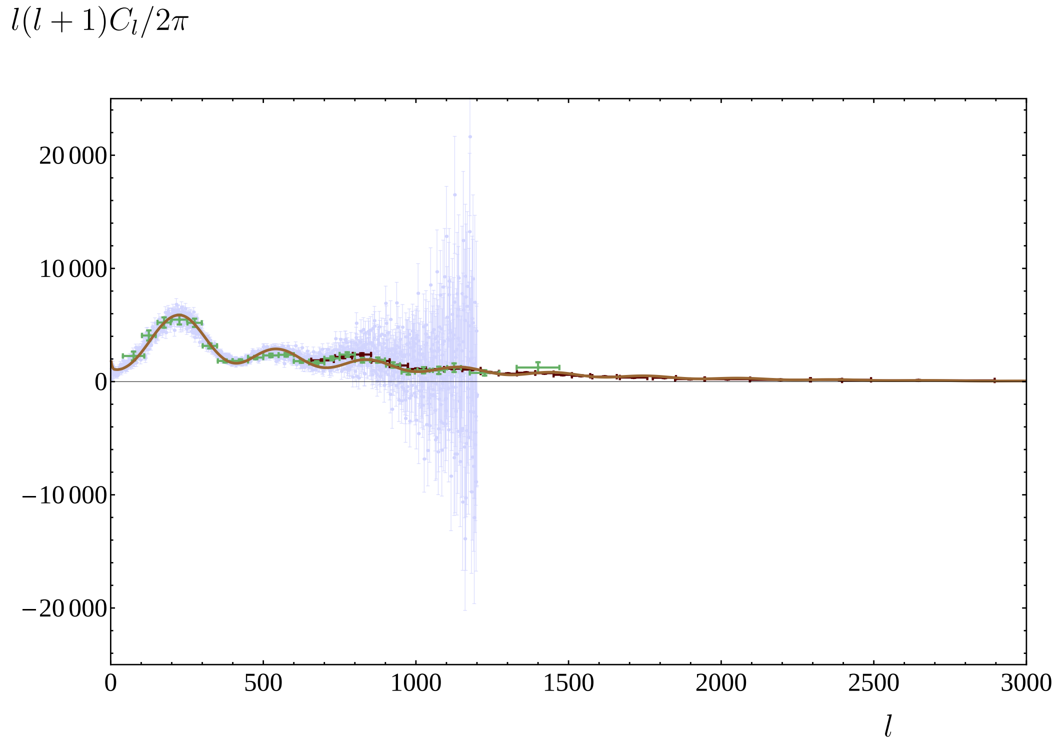

Figure 4. The same as in Fig. 3 with addition of BOOMERANG data [30, 31] (light crosses) and ACBAR data [32] (dark crosses).

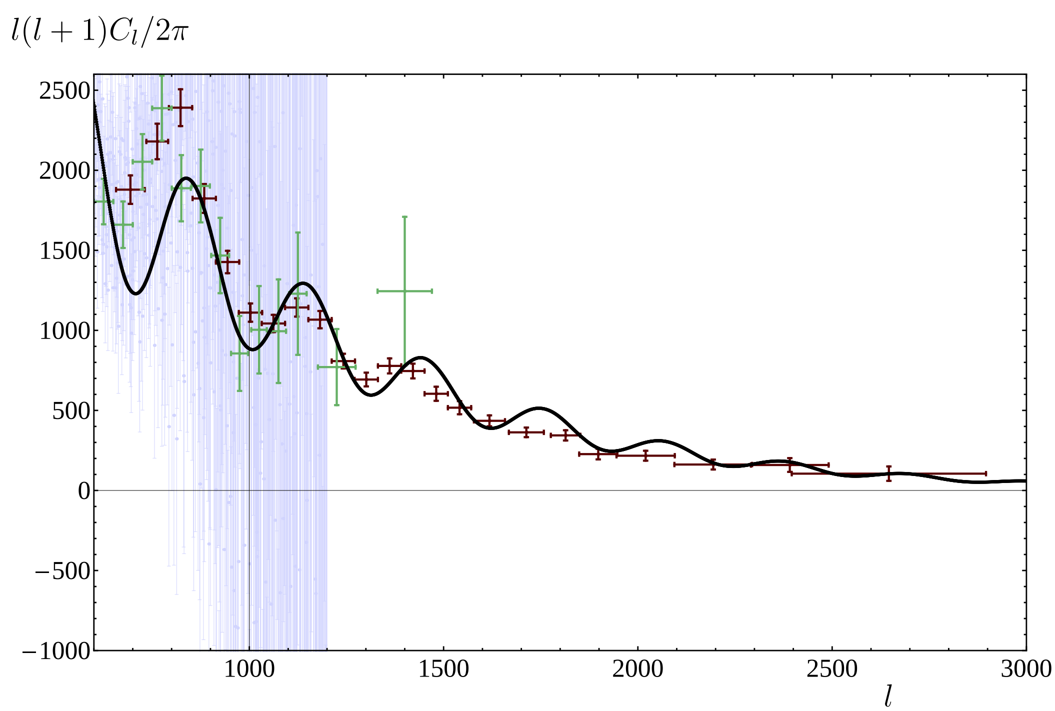

Figure 5. The same as in Fig. 4, but focused at the region of third and further peaks ($l > 600$).

which is just 0.19 worse than the typical value of χ^2/d.o.f. in the concordance model of ΛCDM. Fig. 4 clearly shows that at $l > 600$ the WMAP data suffer from huge statistical and systematical uncertainties. Indeed, the inclusion or exclusion of data at $l > 600$, in practice, do not change χ^2/d.o.f. Therefore, the WMAP data beyond the first and second peaks are not conclusive, at all to the moment.

Then, higher multipoles can be studied due to the BOOMERANG and ACBAR data sets [30, 31, 32]. We present the comparison in Fig. 5. Evidently, the dark matter indication is reduced to the different form of third peak rise. Therefore, we can hope that a nonlinear smearing of sound waves in MOND could give the same effect in the region of third peak, too, although to the moment we see the clear tension between the data in the region of third acoustic peak and the simplest version of fit based on the modified cosmology of homogeneous Universe only. Thus, we need the development of procedure to calculate the transfer function of inhomogeneity versus the red shift [33] in the case of nonlinear MOND. The same is true as concerns for the simulation of large scale structure formation visible at present time.

Nevertheless, we insist on quite the successful fitting of CMBR anisotropy in the framework of cosmological extrapolation of MOND, as the variant of modified gravity involving the critical acceleration in the case of homogeneous matter.

3. Main problems

The enhanced value of baryon fraction in the energy budget, $\Omega_b \approx 0.115$ implies the enhanced value of baryon-to-photon ratio η_b being the only free parameter of BBN calculation [17]. The BBN takes place during the period when the deviations from the general relativity are negligible. Therefore, the primary abundance of light elements in cosmological extrapolation of MOND should differ from the BBN estimated within the concordance model of ΛCDM.

At present the data on the helium abundance has large uncertainties which are compatible with both models under consideration. Then, the deuterium and lithium primary abundances are clearly able to discriminate between the enhanced value of η_b and its concordance value. However, these quantities are not measured directly, they are extrapolations from a suggested model of evolution.

Indeed, we observe the visible sources of light, that mean the primary matter is contracted in stars with further development of nuclear reactions, and a model of star evolution has to reproduce the primary values of abundances. In this respect, one has to take into account the different red shifts of star formation, i.e. different ages of luminous objects, as considered in the framework of general relativity or MOND, of course. Next, at present the standard model for the extrapolation to primary abundances is not self consistent, because it predicts different ratios of both deuterium to lithium-7 and lithium-7 to lithium-6, whereas the former ratio is in bright tension of prediction with the extracted values, while the later is in a deep contradiction (about three order of magnitude!). Thus, the present BBN status can not be surely conclusive.

It is desirable to get more better reliability of procedure for the empirical extraction of primary abundance of light elements.

On the other hand, the doubling of baryonic fraction should appear in cosmological effects. So, this doubling could appropriately explain a missing mass in galactic clusters, as found even within MOND. Then, a significant portion of baryonic matter should be in cold form (Jupiter-like objects), for instance. Note that even in ΛCDM the visible matter composes the tenth fraction of all baryonic matter, only, hence, the most of baryons are in cold form.

Next, the visible large scale structure should be explained by appropriate propagation of primary spatial inhomogeneities. However, this issue of modified gravity involving the critical acceleration is not yet developed because of nonlinearity of the problem.

4. Discussion and conclusion

In this paper, we have shown that the cosmological extrapolation of MOND as the modified gravity involving the critical acceleration, can successfully reproduce main features of multipole spectrum of CMBR anisotropy.

Let us list conclusions of our investigation.

(i) The modified dynamics adjusted to empirical values of the Hubble rate, sound horizon in the baryon-photon medium, acoustic scale in the multipole spectrum of CMBR anisotropy and magnitudes of type Ia supernovae at red shifts $z < 2$, mimics the dark energy with the very specific *equation of state* w_X, shown in Fig. 2. This is the *falsifiable* prediction of cosmological extrapolation of MOND. It can be verified in the nearest future by extensive measuring of type Ia supernova magnitudes versus the red shift. Even $z < 2$, i.e. the scale factor variation within the interval $0.3 < a < 1$, would be enough in order to make decision on the direct falsification of cosmological model involving the critical acceleration. Moreover, such the exotic behavior of dark energy state parameter w_X, if would be confirmed, will be marginally artificial for the general relativity, that would mean the straightforward indication of its inadequateness.

(ii) The *spectral index* of primary spatial inhomogeneity $n_s(k) = n_s^{(0)} + n_s' \ln k/k_0$, essentially runs.

- The character of running is *model-dependent*, and
- It is very different in the concordance model of general relativity and in the cosmological extrapolation of MOND: in the modified gravity the running is rather fast, and its parameters signalize on the hybrid (multified) inflation, that could generate such the spectrum, while the general relativity gives the slow running, which preferably corresponds to the simplest top-hill inflation due to a single inflaton field [34].

- The modified gravity gives $n_s^{(0)} > 1$ at $n_s' \approx \frac{1}{4} > 0$, while the general relativity results in $n_s^{(0)} < 1$ at $n_s' \to 0$.
- The fast running probably indicates the need to improve the calculation tool in order to include higher derivatives of spectral index with respect to logarithm of wave vector.

(iii) The spectral index of primary inhomogeneity define initial conditions for the transfer of inhomogeneity during the evolution, which can be observed in baryonic acoustic oscillations in large scale structure of present Universe [19]. The procedure of calculating the transfer function in the framework of modified gravity with the critical acceleration is nonlinear, and it is not still developed, that does not allow us to make a comparison with data, at present. The structure growth is enhanced in MOND, and it can probably need for additional mechanism of smearing the acoustic oscillations [4].

(iv) The modified gravity in our version results in doubly enhanced fraction of baryons, approximately. This means that baryon-to-photon ratio is twice large, at least.

- More reliable estimates of primary *abundance of light elements* is required in order to *discriminate* the general relativity from the modified gravity by the big bang nucleosynthesis. Therefore, *BBN* can give the *falsification* of cosmological extrapolation of MOND.
- The doubling of baryons in the form of cold baryonic matter (Jupiter-like objects, for instance) should be found in observations. For instance, the mass deficit in galaxy clusters described within MOND, can signalize on the appropriate enhancement of baryons.

(v) The multipole spectrum of CMBR anisotropy needs improvements of accuracy in the range of third acoustic peak. If the model of modified gravity will still miss the correct description of third peak after such the improvement, then this would point to the extension of simplest consideration by strict inclusion of inhomogeneity propagation within the modified dynamics.

Finally, we have shown that the coincidence problem of general relativity is inherently solved in the framework of cosmological extrapolation of MOND: the critical acceleration is connected to the extra Ricci tensor of de Sitter space, involved in the gravity equations; then, it is naturally correlates with the cosmological constant. In addition, the modified gravity is mostly effective at zero acceleration of Universe expansion. That is why the coincidence notion is actual at present.

Keeping in mind soluble problems mentioned, we state that the cosmological extrapolation of MOND is quite successful in cosmology. Moreover, we can falsify it in the nearest future, although the same note on the verification is actual for the general relativity, too.

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